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Progressive optimization on unstructured grids using multigrid-aided finite-difference sensitivities

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SUMMARY

This paper proposes an efficient and robust progressive-optimization procedure, employing cheap, flexible and easy-to-program multigrid-aided finite-differences for the computation of the sensitivity derivatives. The entire approach is combined with an upwind finite-volume method for the Euler and the Navier–Stokes equations on cell-vertex unstructured (triangular) grids, and validated versus the inverse design of an airfoil, under inviscid (subsonic and transonic) and laminar flow conditions. The methodology turns out to be robust and highly efficient, the converged design optimization being obtained in a computational time equal to that required by 11–17 (depending on the application) multigrid flow analyses on the finest grid. Copyright © 2004 John Wiley & Sons, Ltd.

KEY WORDS: optimization; inverse design; multigrid

1. INTRODUCTION

In the last years, many CFD researchers have devoted their efforts to the development of robust and efficient gradient-based optimization procedures for the automatic design of fluid-dynamic components. The most ambitious technique developed so far is the so-called *one-shot* method [1, 2], which combines the objective function and the governing equations, so as to define and solve a unique problem. The alternative, iterative formulation of the optimization problem consists of computing the flow on a trial geometry, evaluating the objective function gradient, and accordingly modifying the shape. Concerning its efficiency, a progressive optimization strategy has been proposed in Reference [3], based on the simultaneous convergence of the

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design process and of all iterative solutions involved (flow analysis, gradient evaluation), also including the global refinement from a coarse to a sufficiently fine mesh. Greater advantage has been taken from the use of multiple grid levels by the methods proposed in References [1, 4, 5], where the descent algorithm and the design variables work according to multigrid [6] concepts.

Perturbed shapes and finite differences can be used to evaluate the sensitivities. This method is easy-to-program, is invariant with respect to the grid type, to the flow modelization and to the discretization scheme, and can be combined with black-box commercial codes. Its drawback is the large amount of computational work, almost proportional to the number of design parameters. To overcome this low efficiency, adjoint methods, in both continuous and discrete form, have been developed and widely tested, see, e.g. Reference [7]: all sensitivities are computed by solving a unique adjoint system, independent of the number of design parameters. This approach is very efficient, but a very cumbersome, analytical or symbolic, differentiation of the governing equations is required. An alternative, much simpler, approach has been proposed in Reference [8], where it is shown that when the objective function depends on surface integrals, its gradient is not significantly influenced by the flow derivatives, which can be dropped out. The cost of the gradient evaluation becomes negligible, but all applications proposed so far lead to an improvement, rather to a complete optimization, of the performances of the component under design. An alternative approach, which also takes advantage of the multigrid concepts, but differently from References [1, 4, 5], has been recently proposed by the authors [9]: this Multigrid-Aided Finite-Difference (MAFD) technique should be invariant with respect to the grid type, to the flow modelization and to the discretization scheme, as the standard finite-difference approach; however, to date, it has been tested, in combination with the progressive optimization strategy of Reference [3], only versus the inviscid flow past a 3D turbine nozzle in inviscid transonic flow conditions, using a structured, cell-centred, flow solver. The aim of this paper is to demonstrate the invariance cited above, by combining the MAFD progressive optimization technique with a different, cell-vertex, unstructured discretization [10], and by extending its application to a different flow model, namely to laminar flow conditions.

2. FLOW SOLVER

An unstructured cell-vertex triangular grid is used to discretize the 2D Euler and Navier–Stokes equations governing the flows considered in this paper. A left state and a right state are linearly reconstructed on the two sides of each interface (ij), obtained by connecting either the barycentres or the circumcentres of two neighbouring triangles. Similar to the 1D case, a unique left-neighbouring cell is used to define the flow gradient employed in the reconstruction [10]: it is defined as the cell C_{ji} which contains the prolongation of the side (ji), plotted as a dot–dashed line in Figure 1. Standard one-dimensional limiters are also applied straightforwardly. The flux-difference-splitting of Roe [11] is then used to solve the Riemann problem defined at each interface. A standard finite-element Galerkin discretization is used for the viscous terms.

The discretized governing equations are solved by means of a four-stage Runge–Kutta scheme, coupled with an Implicit Residual Smoothing procedure: Reference [12] fully describes the technique here employed to define the smoothing lines on cell-vertex unstructured grids.

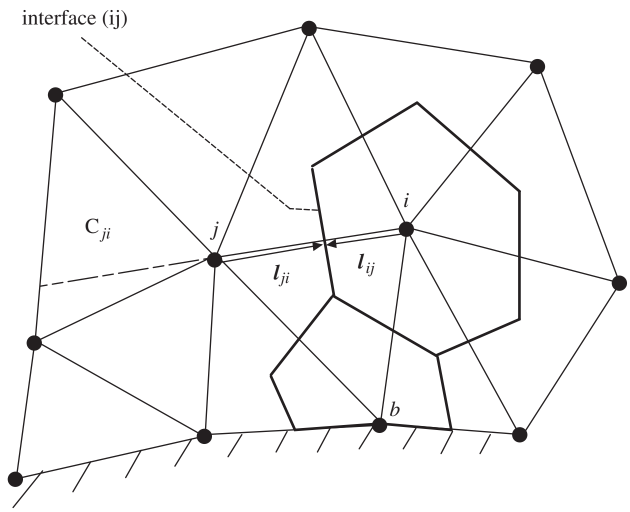

Figure 1. Higher-order reconstruction.

A standard V-cycle full multigrid (FMG) [6] has been also implemented both to accelerate convergence to steady state and to compute the MAFD sensitivities. Finer grids are created during the nested iteration by means of a global uniform refinement, improved by a grid-point adjustment.

3. PROGRESSIVE OPTIMIZATION

The simultaneous convergence of the design process and of the flow analysis, also including the global refinement from a coarse to a sufficiently fine mesh, is the basis of the progressive optimization strategy proposed in Reference [3] and here employed: less accurate sensitivity derivatives (i.e. with partially converged flow solutions computed on coarser levels) are used when the geometry is far from the optimal one; then, the convergence level of the flow solution and the number of mesh points are increased while approaching the optimum. Starting the optimization on coarser grids and using partially converged flow solutions drastically reduce the computational cost of the entire optimization procedure, without affecting its robustness and capability of finding the optimum, as demonstrated by the large number of applications proposed so far. Full details can be found in References [3, 9].

4. MULTIGRID-AIDED FINITE-DIFFERENCE SENSITIVITIES

The MAFD procedure proposed in this paper aims at reducing the computational work required by the flow computation on perturbed geometries, while maintaining the advantages cited in Section 1. According to the so-called *dual viewpoint* of the multigrid (MG) technique, a constant term is computed at each MG cycle and added to the right-hand side on the coarser grid level: as known, this term represents an approximate value of the relative local truncation error (RLTE) between the finer grid and the coarser one [6]. Accordingly, the MAFD method is based on the following important considerations: (i) the MG strategy solves the flow equations on coarser grid levels with the same accuracy of the finer level, thanks to the addition of the RLTE term. Moreover, (ii) a correct choice of the design parameters should give a smooth perturbation of the blade profile, that can be seen effectively on a coarser level. Finally, (iii) the approximate RLTE, which mainly represents the difference of accuracy between two nested grid levels, is not affected by a small, smooth perturbation of one design parameter. On the basis of these three considerations, the proposed method allows to compute the difference between the flow solutions of two perturbed geometries using a coarser grid level and a value of the RLTE computed only once, using the unperturbed geometry. Centred finite differences have been preferred with respect to one-side differences for robustness, rather than for accuracy. However, an exhaustive comparison between the two possible approaches has not been performed yet. It is noteworthy that almost identical performances are obtained with a step-size $\Delta\xi$ ranging from 10^{-3} to 10^{-5}, provided that the convergence level ($\log_{10}$ of the flow residual) of the coarse-grid solution for the perturbed shapes p, is related to the current convergence level of the finest grid solution f, by the following empirical relation:

$$p = f - 1 - 0.5(3 + \log_{10}\Delta\xi) \tag{1}$$

It is noteworthy that the coarse-grid evaluation of the perturbed flow fields is very efficient, while preserving the fine-grid accuracy: the coarser grid levels have a much lower number of cells and allow the use of a higher time step (the time step is at least doubled at each coarsening). The required computational work still depends on the number of design parameters N_ξ, but has been drastically reduced.

5. RESULTS

The inverse design of an airfoil, in both inviscid (subsonic and transonic) and laminar flow conditions, has been approached in this paper to validate the proposed strategy, namely to check its capability to reach the optimum and its efficiency: the target shape, defined by known values of the design parameters ξ_j, is used to compute a target pressure distribution, $\hat{p}$, that must be matched by the pressure distribution p, computed on the current trial shape: accordingly, the objective function is defined as

$$I(\boldsymbol{\xi}) = \frac{1}{2S}\int_S [p - \hat{p}]^2 \, \mathrm{d}S \tag{2}$$

All applications have been obtained with the same code, except for the addition of a simple high-frequency filtering (smoothing) of the target and of the computed pressure distributions, when evaluating the sensitivities in the transonic flow case, where a sharp shock is captured by the flow solver. The known target airfoil has been preliminarily defined by combining the four orthogonal (two symmetric and two antisymmetric) base functions [9] provided in Figure 2, with weights (or design parameters) $\xi_1 = 1.2$, $\xi_2 = 0.5$, $\xi_3 = 0.1$ and $\xi_4 = 0.1$, see

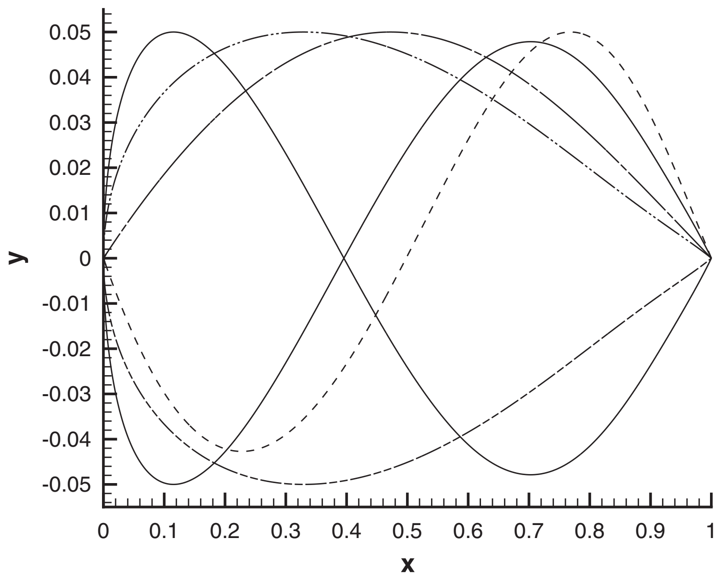

Figure 2. Orthogonal base functions.

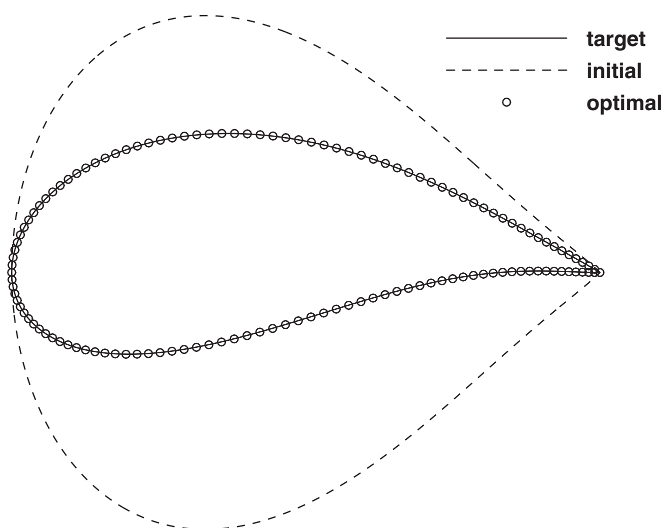

Figure 3. Target, initial and optimal profiles.

the solid line plotted in Figure 3. In all cases, the initial profile, plotted as a dashed line in Figure 3, is defined by the design parameters $\xi_1 = 3$, $\xi_2 = 0$, $\xi_3 = 0$ and $\xi_4 = 0$. Both the MG and the progressive optimization employ three grid levels, the finest mesh being com-

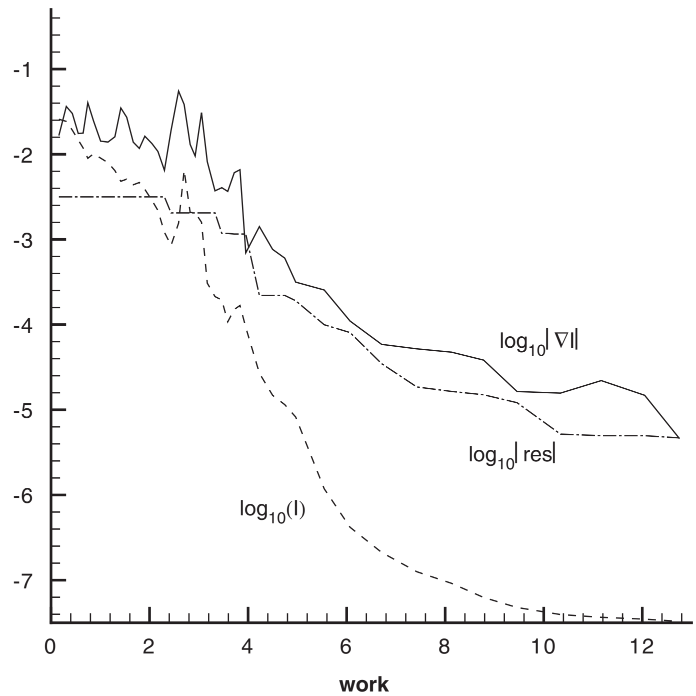

Figure 4. Convergence histories (inviscid subsonic).

posed of 11 734 nodes and 22 960 triangles (392 nodes on the airfoil). The two grid refinements have been always set for $\log_{10}|\nabla I| \leqslant -2.5$ and $\log_{10}|\nabla I| \leqslant -3.5$, respectively. The whole optimization process is stopped when $\log_{10}|\nabla I| \leqslant -5.0$ at the finest level, which is even excessive for engineering applications. Both subsonic and transonic inviscid flow conditions have been considered (incidence angle $\alpha = 1^\circ$), with $M_\infty = 0.5$ and 0.8, respectively. Figures 4 and 5 propose the convergence histories of the flow residual, of the objective function and of the magnitude of the objective function gradient, for the subsonic and the transonic regimes, respectively. One work unit is defined as the computational time required to run one converged MG analysis of the target airfoil on the finest mesh.

The optimization procedure is firstly applied on the coarsest level, which cannot take advantage of the MAFD procedure. In the subsonic (transonic) test case, the first grid refinement is performed after 4 (0.8) work units. At the second grid level, the MAFD technique employs the coarser mesh to compute the sensitivity derivatives: the second refinement is then located at work ≈ 5 (work ≈ 1.6). Clearly, the work spent on the coarser levels becomes more relevant with respect to previous applications, since the advantages of the coarse-grid evaluation of the perturbed flow-fields are reduced or even missed. Figures 4 and 5 indicate that the work required to obtain the more than satisfactory convergence level of -5.0, on the finest mesh, is about 11–13. This convergence level is even excessive for engineering applications: for the transonic test-case, for example, the optimization has found, at this convergence level, the optimal design parameters $\xi_1 = 1.2006$, $\xi_2 = 0.5005$, $\xi_3 = 0.1000$ and $\xi_4 = 0.0997$; the optimal airfoil (symbols) is perfectly superposed to the target configuration (solid line), see

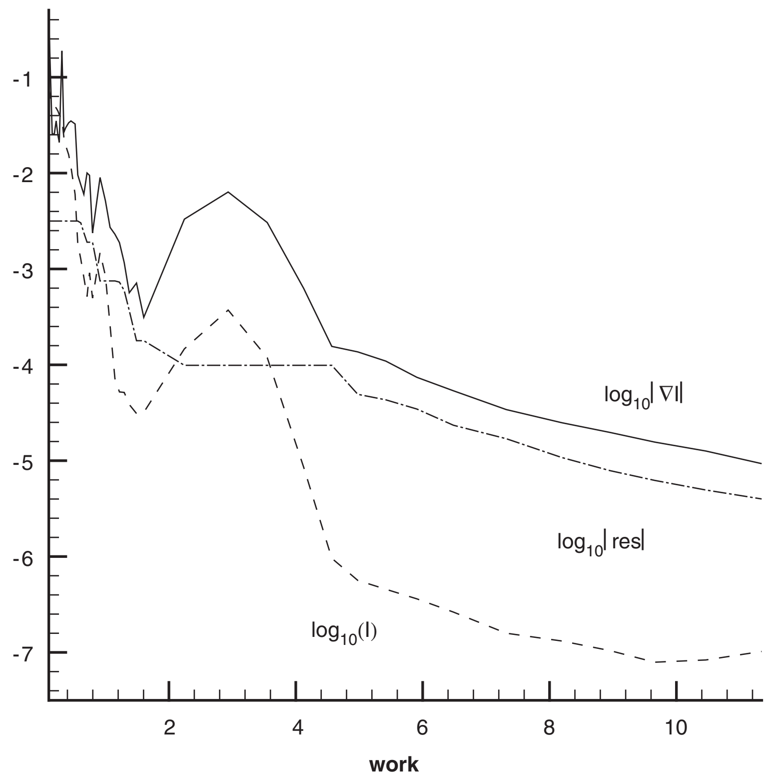

Figure 5. Convergence histories (inviscid transonic).

Figure 3. Incidentally, the comparison with *almost-exact* sensitivities, computed by using finite differences on the finest grid with well-converged perturbed flow solutions and different $\Delta\xi$, provides a discrepancy usually ranging from 3 to 10%, with rare higher values, which however never exceed 20%. This discrepancy is due to the MAFD approximation as well as to the incomplete convergence of both the perturbed and the finest-grid flow solutions.

The MAFD progressive optimization has been finally tested using the same target and initial airfoils under laminar flow conditions, with separation ($M_\infty = 0.8$, $Re = 500$, incidence angle $\alpha = 10°$): Figure 6 provides the convergence histories of the optimization. The two grid refinements are performed after 3 and 4 work units, respectively. Figure 6 indicates that the work required to obtain the fully satisfactory convergence level of -5.0, on the finest mesh, is about 17.

6. CONCLUSIONS

A very efficient and robust progressive-optimization procedure using MAFD for the computation of the sensitivity derivatives has been proposed and combined with an upwind finite-volume method for the Euler and the Navier–Stokes equations on cell-vertex unstructured

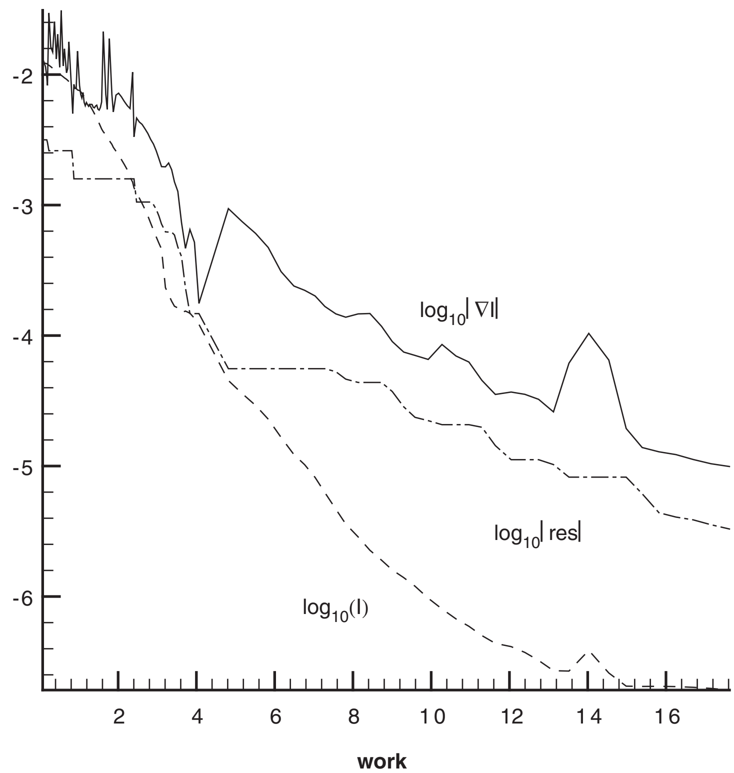

Figure 6. Convergence histories (laminar).

triangular grids. The entire approach is cheap, flexible and easy-to-program; it also turns out to be robust and highly efficient, the converged design of an airfoil, under inviscid (subsonic and transonic) and laminar flow conditions, being obtained in a computational time equal to that required by 11–17 multigrid flow analyses on the finest grid.

REFERENCES

1. Kuruvila G, Ta'asan S, Salas MD. Airfoil desing and optimization by the one-shot method. *AIAA Paper 95-0478*, 1995.
2. Held C, Dervieux A. One-shot airfoil optimisation without adjoint. *Computers and Fluids* 2001; **31**:1015–1049.
3. Dadone A, Grossman B. Progressive optimization of inverse fluid dynamic design problems. *Computers and Fluids* 2000; **29**:1–32.
4. Mohammadi B, Pironneau O. Applied shape optimization for fluids. *Oxford Science Publications*. Clarendon Press: Oxford, 2001.
5. Beux F, Dervieux A. A hierarchical approach for shape optimization. *Engineering Computations* 1994; **11**: 25–48.
6. Brandt A. Multi-level adaptive solutions to boundary-value problems. *Mathematics of Computation* 1977; **31**:333–390.
7. Jameson A. Aerodynamic design via control theory. *SIAM Journal on Scientific Computing* 1988; **3**:233–260.
8. Mohammadi B. A new optimal shape design procedure for inviscid and viscous turbulent flows. *International Journal for Numerical Methods in Fluids* 1997; **25**:183–203.
9. Catalano LA, Dadone A, Daloiso VSE. Progressive optimization using orthogonal shape functions and efficient finite-difference sensitivities. *AIAA Paper 2003-3961*, 2003.

10. Catalano LA. A new reconstruction scheme for the computation of inviscid compressible flows on 3D unstructured grids. *International Journal for Numerical Methods in Fluids* 2002; **40**(1–2):273–279.
11. Roe PL. Characteristic based schemes for the Euler equations. *Annual Review of Fluid Mechanics* 1986; **18**:337–365.
12. Catalano LA, Daloiso VSE. Upwinding and implicit residual smoothing on cell-vertex unstructured grids. *International Journal for Numerical Methods in Fluids* 2005, to appear.

